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MOTION-RELATED BODY-FORCE FUNCTIONS IN TWO-DIMENSIONAL LOW-SPEED FLOW

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The paper examines two-dimensional force functions useful in the wind-response analysis of bluff bodies, such as bridges, that are elongated in the across-wind direction. Airfoil-type theoretical 2-D indicial, admittance, and oscillatory force functions and their interrelationships are first recalled for their analogical value. Following this, a spectral force expression is developed for the lift on a bluff section due to a cross-wind with a vertical turbulence component. The expression proposed involves an aerodynamic admittance that is based upon measured flutter derivatives for the bluff section plus information on the coherence of vertical turbulence in the atmospheric boundary layer. © 2000 Academic Press

1. INTRODUCTION

THE PRESENT PAPER is part of the continuing search to improve analytic models for wind forces on structures of extended span, such as bridges. The cross-sections of such structures must usually be considered bluff in form, although some modern bridges have cross-sections that are quite streamlined.

While all physical flows are inherently three-dimensional, the two-dimensional or sectional point of view has historically proven its value as a step toward understanding the central characteristics of many practical problems of fluid–structure interaction. This viewpoint is retained in the present paper, since it deals with along-wind sections of structures that are considerably extended in the across-flow direction. It is akin to the case of infinite aspect ratio in aeronautical parlance.

For a body placed in a cross-flow, the flow-induced forces developed on it will in general be time-dependent as a consequence of body shape or motion, or of turbulence in the approach flow. Analytical force models in this context, created for the purposes of structural design, thus require formats that permit quantification of their frequency and amplitude dependencies. This paper will review some models of this kind, characterizing them according to time dependence and both complex-Fourier and power-spectral frequency dependence. Some classical airfoil results will first be reviewed as possible procedural prototypes. The end objective of the present paper is, through a review of some models of lift behavior, to arrive at a practical one that incorporates available dynamic, experimental evidence measured on the particular body under study.

When there occurs an abrupt change in the attitude of the body relative to the horizontal cross-flow, a transient is initiated in the associated force acting on the body. This typically is not instantaneous but takes some time to develop. Some particular fluid-body relative motions that have "indicial" or unitary characteristics form a basis for the study of the ensuing force transients. Typical of these are: (a) a step function change in the effective angle

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of attack of the flow relative to the body; (b) penetration of the body into (or envelopment of the body by) a half-space endowed with a specified vertical velocity; and (c) oscillatory motion of the body relative to its rest position.

A simple monotonic function describing an elementary evolution from a fractional starting value to a unitary terminal value ($s \to \infty$) is the function

$$\Phi(s) = 1 - a\mathrm{e}^{-bs}, \tag{1}$$

where s is a variable like time and a, b are constants. This can be given more descriptive flexibility by the inclusion of more exponential terms. In airfoil transient-lift theory the evolutionary form

$$\Phi(s) = 1 - a\mathrm{e}^{-bs} - c\mathrm{e}^{-ds}, \tag{2}$$

where s is the dimensionless time and a, b, c and d are constants, has proven useful. This function which, historically, has been used in many applications, can take on a variety of forms depending on the values chosen for the four parameters a, b, c *and* d. Several alternate choices of parameters will be employed in what follows.

2. CLASSICAL THIN AIRFOIL INDICIAL LIFT

Steady-state or mean vertical lift† L on an airfoil is described by the formula

$$L = \tfrac{1}{2}\rho U^2 B C_L, \tag{3}$$

where ρ is air density, U is cross-flow velocity, B is airfoil chord, and C_L is an appropriate constant lift coefficient. If C_L is fixed at a value C_{L_0} and the angle of attack of the airfoil is changed by a small amount α, the steady vertical lift becomes, to a common first approximation (Maclaurin expansion):

$$L = \tfrac{1}{2}\rho U^2 B[C_{L_0} + C'_L \alpha], \tag{4}$$

where $C'_L = \mathrm{d}C_L/\mathrm{d}\alpha$ at that value of C_{L_0}.

If, on the other hand, the steady-state condition is altered by an abrupt step-function change from a zero-lift condition to an incremental angle of attack α_0, the lift undergoes a transient change described by

$$L(s) = \tfrac{1}{2}\rho U^2 B C'_L \alpha_0 \varphi(s), \tag{5}$$

where $s = 2Ut/B$ is a dimensionless time, or distance expressed in chord lengths, and $\varphi(s)$ is an *indicial lift-growth function*. The form of this function was determined theoretically for a thin airfoil (flat plate) by Wagner (1925). The function is pictured in Figure 1. Its limiting characteristics were determined by Wagner to be

$$\varphi(0) = 0{\cdot}5 \quad \text{and} \quad \lim_{s\to\infty} \varphi(s) = 1.$$

It follows that, if linear superpositional principles are invoked, the time-history of lift associated with an arbitrary small airfoil motion $\alpha(s)$ can be formally expressed as

$$L(s) = \frac{1}{2}\rho U^2 B C'_L \int_{-\infty}^{s} \varphi(s-\sigma)\,\alpha'(\sigma)\,\mathrm{d}\sigma, \tag{6}$$

† Throughout the present context, lift L is assumed to be *vertical* rather than normal to the relative wind as in the conventional aeronautical context.

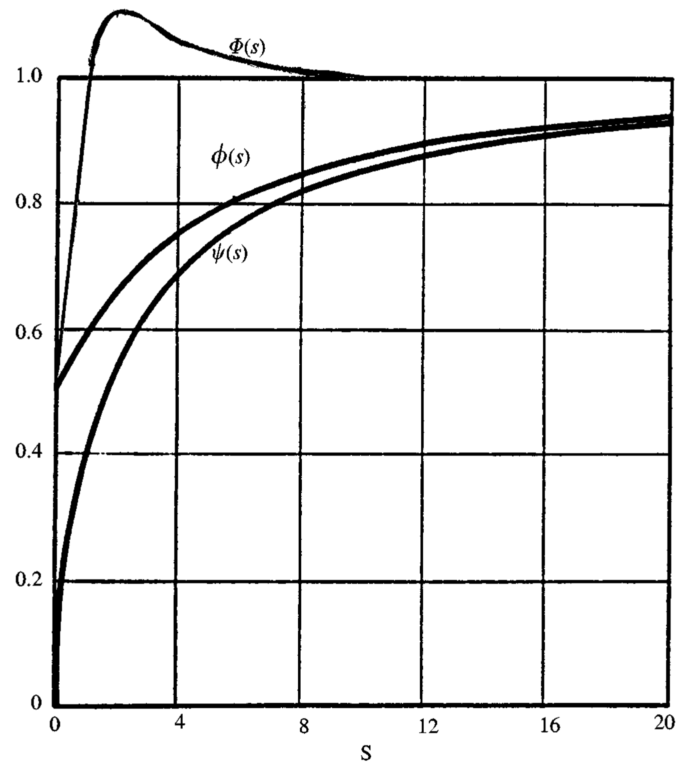

Figure 1. Indicial response functions: Wagner, Küssner and representative bluff body.

or, by a change of variables and integration by parts,

$$L(s) = \frac{1}{2}\rho U^2 B C_L' \left\{ \varphi(0)\,\alpha(s) + \int_0^{\infty} \varphi'(\sigma)\,\alpha(s-\sigma)\,\mathrm{d}\sigma \right\}. \tag{7}$$

Jones (1940) offered an excellent approximation for $\varphi(s)$, namely the form (2) in which $a = 0{\cdot}165$, $b = 0{\cdot}0455$, $c = 0{\cdot}335$ and $d = 0{\cdot}300$. Its first derivative is

$$\varphi'(s) = a\,b\,\mathrm{e}^{-bs} + c\,d\,\mathrm{e}^{-ds}. \tag{8}$$

3. CLASSICAL AIRFOIL FLUTTER

Theodorsen (1934) developed complex expressions for the oscillating lift and moment on a thin airfoil undergoing complex vertical (h) and torsional (α) oscillatory motion:

$$h = h_0 \mathrm{e}^{\mathrm{i}\omega t} = h_0 \mathrm{e}^{iks}, \quad \alpha = \alpha_0 \mathrm{e}^{\mathrm{i}\omega t} = \alpha_0 \mathrm{e}^{iks}$$

where k is the dimensionless oscillation frequency $k = B\omega/2U$ and ω is the circular oscillation frequency:

$$L = -\frac{\rho B^2}{4}\left(\pi U\dot{\alpha} + \pi\ddot{h} - \frac{\pi B}{2}a\ddot{\alpha}\right) - \pi\rho U B C(k)\left[U\alpha + \dot{h} + \frac{B}{2}\left(\frac{1}{2} - a\right)\dot{\alpha}\right], \tag{9}$$

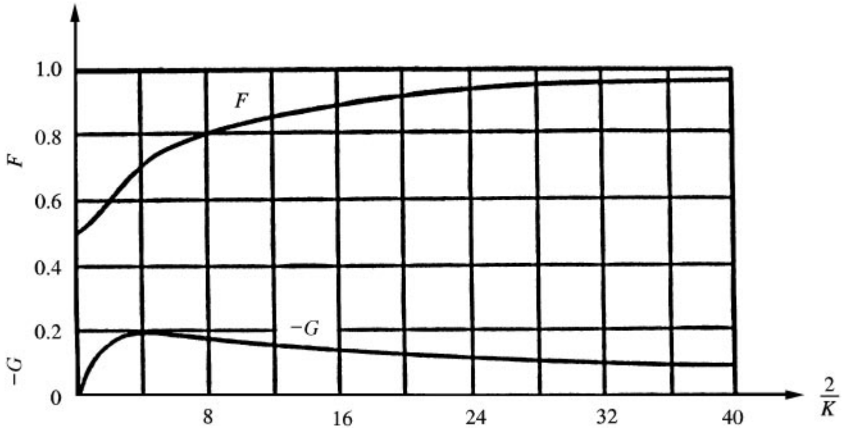

Figure 2. Real and imaginary components of Theodorsen circulation function.

$$M = -\frac{\rho B^2}{4}\left\{\pi\left(\frac{1}{2}-a\right)U\frac{B}{2}\dot{\alpha}+\frac{\pi B^2}{4}\left(\frac{1}{8}+a^2\right)\ddot{\alpha}+\frac{\pi a B}{2}\ddot{h}\right\}$$
$$+\frac{\rho U B^2}{2}\pi\left(a+\frac{1}{2}\right)C(k)\left[U\alpha+\dot{h}+\frac{B}{2}\left(\frac{1}{2}-a\right)\dot{\alpha}\right]. \quad (10)$$

In equations (9) and (10) the theoretical lift curve slope is $C_L' = 2\pi$; $aB/2$ is the distance from the airfoil midchord to the oscillatory rotation point; h is the vertical deflection of that point; α is the rotational angle about it; and the function $C(k)$ is the renowned Theodorsen (1934) circulation function

$$C(k) = F(k) + \mathrm{i}G(k), \quad (11)$$

originally expressed in terms of Bessel or Hankel functions. The functions F and G have the forms plotted in Figure 2. The terms in equations (9) and (10) with the factor $C(k)$ constitute the principal, i.e. "circulatory", part of the lift or moment. It is noteworthy that, while (the oscillatory) L and M are expressed in the time domain, the coefficients dependent on $C(k)$ in equations (9) and (10) are defined in the frequency domain. This is a built-in characteristic of the classical flutter theory. An interesting further result for the circulatory terms of equations (9) and (10) is that they depend on the effective vertical velocity of the rearward 3/4 chord point of the airfoil.

For an oscillatory motion consisting only of the vertical velocity $\dot{h}$ the circulatory lift according to equation (9) is

$$L = -\pi\rho U B C(k)\dot{h}, \quad (12)$$

the value of which may be rewritten more conventionally as

$$L = \tfrac{1}{2}\rho U^2 B C_L' \chi_T(k)\alpha, \quad (13)$$

where $\alpha = \dot{h}/U$, $C_L' = 2\pi$ and the factor $\chi_T(k)$ can be recognized as a *complex aerodynamic admittance* function

$$\chi_T(k) = F(k) + \mathrm{i}G(k). \quad (14)$$

According to the classical thin airfoil theory, the lift (13) acts at the airfoil forward quarter-chord point. If expression (13) is compared to equation (7) for $\alpha = \dot{h}/U = \alpha_0 e^{iks}$, the following equivalence can be demonstrated:

$$C(k) = \varphi(0) + \overline{\varphi'}, \tag{15}$$

where

$$\overline{\varphi'}(k) = \int_0^\infty \varphi'(\sigma)\, e^{-ik\sigma}\, d\sigma \tag{16}$$

is the Fourier transform of the first s-derivative of the Wagner function $\varphi(s)$. The relation (15) was first demonstrated by Garrick (1938).

From equations (15) and (8), and noting that the Fourier transform of e^{-bs} is $1/(b + ik)$, the following excellent approximations may be derived for the functions $F(k)$ and $G(k)$:

$$F(k) = 1 - a - c + \frac{ab^2}{b^2 + k^2} + \frac{cd^2}{d^2 + k^2}, \tag{17}$$

$$G(k) = -k\left[\frac{ab}{b^2 + k^2} + \frac{cd}{d^2 + k^2}\right], \tag{18}$$

with a, b, c and d as specified earlier.

The power spectrum $S_L(k)$ of the lift (13) is thus related to that of the angle of attack $S_\alpha(k)$ by

$$S_L(k) = [\tfrac{1}{2}\rho U^2 B C_L']^2\, |\chi_T(k)|^2 S_\alpha(k), \tag{19}$$

where

$$|\chi_T(k)|^2 = F^2(k) + G^2(k) \tag{20}$$

is the squared amplitude of the *Theodorsen aerodynamic admittance function*. This function has the form plotted in Figure 3.

The relations reviewed to this point are theoretical circulatory lift functions of thin airfoil vertical motion relative to a smooth horizontal approach flow. Analogous observations will next be noted for cases in which the motion is that of the flow moving vertically relative to the body.

4. AIRFOIL PENETRATION OF A UNIFORM GUST

Küssner (1936) analyzed the situation in which a thin airfoil moving at horizontal velocity U penetrates a half-space of uniform vertical velocity w_0, where w_0 is small relative to U. The resulting lift at the forward quarter-chord of the airfoil takes the form

$$L(s) = \tfrac{1}{2}\rho U B C_L' w_0 \psi(s), \tag{21}$$

where the function $\psi(s)$ evolves with s as depicted in Figure 1. This theoretical function has the characteristics that

$$\psi(0) = 0 \quad \text{and} \quad \lim_{s\to\infty} \psi(s) = 1.$$

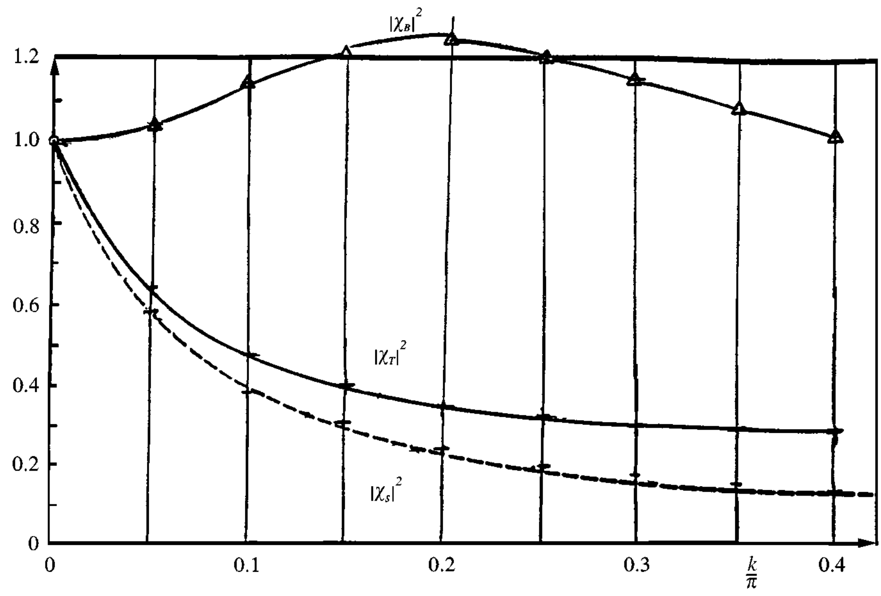

Figure 3. Admittance functions: $|\chi_T|^2$ = Theodorsen; $|\chi_S|^2$ = Sears; $|\chi_B|^2$ = representative bluff body.

By replacing $\Phi(s)$ by $\psi(s)$ in equation (2), an excellent approximation for the function ψ is given by equation (2) if for this case a, b, c and d are given the alternate values

$$a = c = 0{\cdot}5, \quad b = 0{\cdot}130, \quad d = 1{\cdot}00.$$

By superposition, the lift due to the penetration of a gust of any form $w(s)$ of variable gust velocity, may, following the pattern of equation (7), be expressed by

$$L(s) = \frac{1}{2}\rho U B C_L' \left[\psi(0)\, w(s) + \int_0^\infty \psi'(\sigma)\, w(s-\sigma)\, \mathrm{d}\sigma \right]. \tag{22}$$

5. AIRFOIL PENETRATION OF A SINUSOIDAL GUST

Sears (1941) analyzed the situation of a thin airfoil penetrating a half-space in which a vertical gust is undergoing the sinusoidal vertical velocity $w(s) = w_0 \mathrm{e}^{\mathrm{i}wt} = w_0 \mathrm{e}^{\mathrm{i}ks}$. The resulting quarter-chord lift on the airfoil takes the form

$$L(s) = \tfrac{1}{2}\rho U B C_L' \chi_S(k)\, w_0 \mathrm{e}^{\mathrm{i}ks}, \tag{23}$$

in which

$$\chi_S(k) = \psi(0) + \overline{\psi'}(k) = F_S(k) + \mathrm{i}G_S(k) \tag{24}$$

is the complex Sears aerodynamic admittance function, $\overline{\psi'}$ being the Fourier transform of ψ'. Excellent approximations to the functions $F_S(k)$ and $G_S(k)$ can be obtained from equations (17) and (18) by replacing F by F_S, G by G_S, and giving a, b, c and d the values $a = c = 0{\cdot}5$, $b = 0{\cdot}130$, $d = 1{\cdot}0$ used above in the approximation for $\psi(s)$.

The power spectral density of $L(s)$ in this case is

$$S_L(k) = [\tfrac{1}{2}\rho UBC_L']^2\ |\chi_S|^2 S_w, \tag{25}$$

where

$$|\chi_S|^2 = F_S^2(k) + G_S^2(k). \tag{26}$$

In the frequency domain the function $|\chi_S|^2$ is commonly known as the *Sears admittance function* and has the appearance depicted in Figure 3. It serves to link, for an airfoil, the spectrum of vertical gusting velocity to that of associated lift. This function has the convenient empirical approximation (Scanlan 1993):

$$|\chi_S|^2 = \frac{1}{1+5k}. \tag{27}$$

To this point theoretical circulatory motional aerodynamic effects for a thin airfoil have been linked to their corresponding aerodynamic admittances. How results of this type may be accomplished for other than theoretical cases is addressed next.

6. BLUFF-BODY FLUTTER FORCES

For general bodies of nonstreamlined cross-section at which some flow separation typically occurs, no general circulatory theory exists for deriving body-force results analogous to those above from first principles such as potential flow theory. Instead, it has become customary to identify forces associated with bluff-body oscillation by experimental means (Scanlan & Tomko 1971). Over the last two decades, a variety of effective experimental means to this end have evolved. These will not be reviewed here.

Consider a bluff body with degrees of freedom h (vertical) and α (rotation). The following forms have been commonly employed (Scanlan & Jones 1990) to associate motional aerodynamic sectional forces with structural, purely oscillatory, degrees of freedom h and α:

$$L = \frac{1}{2}\rho U^2 B\left[KH_1^*\frac{\dot{h}}{U} + KH_2^*\frac{B\dot{\alpha}}{U} + K^2H_3^*\alpha + K^2H_4^*\frac{h}{B}\right], \tag{28}$$

$$M = \frac{1}{2}\rho U^2 B^2\left[KA_1^*\frac{\dot{h}}{U} + KA_2^*\frac{B\dot{\alpha}}{U} + K^2A_3^*\alpha + K^2A_4^*\frac{h}{B}\right]. \tag{29}$$

In these equations the coefficients $H_i^*(K)$ and $A_i^*(K)$, $i = 1, 2, 3, 4$, are considered to be experimentally determined functions of K; ($K = 2k$), where $K = B\omega/U$, ω being the oscillation circular frequency. For sinusoidal motions, the effects of inertial terms not specifically provided are contained implicitly within the displacement terms. The coefficients $H_i^*(K)$, $A_i^*(K)$, $i = 1, \ldots, 4$, have obvious correspondences to the Theodorsen results in the case of the thin airfoil [cf. equations (9) and (10)]. In some cases of experimental values for H_i^*, A_i^*, apparent or approximate relations between certain pairs of these coefficients have been suggested. Such relations should be viewed as fortuitous rather than causally linked, since, in the case of an arbitrary bluff body, no unifying circulatory function analogous to $C(k)$ exists. On the other hand, an indicial function $\Phi(s)$ based on the full, observed oscillatory forces can be determined. Postulating Φ as such a function, for an arbitrary effective angle of attack $\alpha(s)$, equation (7) gives the lift evolution as

$$L(s) = \frac{1}{2}\rho U^2 BC_L' \left\{ \Phi(0)\,\alpha(s) + \int_0^{\infty} \Phi'(\sigma)\,\alpha(s-\sigma)\,d\sigma \right\}. \tag{30}$$

If now the case is considered where $\alpha(s) = \dot{h}(s)/U$ and α is complex sinusoidal:

$$\alpha(s) = \alpha_0 e^{iks}.$$

The lift $L(s)$ takes the form

$$L(s) = \tfrac{1}{2}\rho U^2 BC_L' \{\Phi(0) + \overline{\Phi'}\}\,\alpha_0 e^{iks}, \tag{31}$$

where $\overline{\Phi'}$ is the Fourier transform of $\Phi'(s)$. The corresponding form for the lift according to equation (28), since $\dot{h} = i\omega h$, would be

$$L(s) = \tfrac{1}{2}\rho U^2 BK[H_1^* - iH_4^*]\,\alpha_0 e^{iks}. \tag{32}$$

Thus, $\Phi(s)$ is linked to the flutter derivatives H_1^* and H_4^* by the relation

$$C_L'[\Phi(0) + \overline{\Phi'}] = K[H_1^* - iH_4^*]. \tag{33}$$

The Fourier transform $\bar{L}$ of the lift convolution $L(s)$ as given by equation (30) is then

$$\begin{aligned} \bar{L}(k) &= \tfrac{1}{2}\rho U^2 BC_L'\{\Phi(0) + \overline{\Phi'}(k)\}\,\bar{\alpha}(k) \\ &= \tfrac{1}{2}\rho U^2 BK[H_1^* - H_4^*]\,\bar{\alpha}(k). \end{aligned} \tag{34}$$

The power spectral density $S_L(k)$ of $L(s)$ is then given by

$$S_L(k) = [\tfrac{1}{2}\rho U^2 B]^2 K^2 [H_1^{*2} + H_4^{*2}] S_\alpha(k). \tag{35}$$

The expression

$$|\chi_B|^2 = K^2[H_1^{*2} + H_4^{*2}]/C_L'^2 \tag{36}$$

may then be recognized as the *aerodynamic admittance* associated with lift due to vertical velocity $\dot{h}(s)$. It is linked to its associated indicial function via the relation

$$|\chi_B|^2 = [\Phi(0) + \overline{\Phi'}][\Phi(0) + \overline{\Phi}'^*], \tag{37}$$

where $\bar{\Phi}'^*$ is the complex conjugate of $\bar{\Phi}'$.

If the detailed form of $\Phi(s)$ is desired it may be inferred from best-fit approximations to experimentally measured functions H_1^* and H_4^*, employing equation (33). A process of this kind was used by Scanlan *et al.* (1974). In a representative case close to one of theirs, the form (2) for Φ corresponding to a step change in vertical velocity of a bluff section may be associated with a set of coefficients like

$$a = -2{\cdot}5,\ b = 0{\cdot}8,\ c = 3{\cdot}0,\ d = 1,$$

i.e.

$$\Phi(s) = 1 + 2{\cdot}5e^{-0{\cdot}8s} - 3{\cdot}00e^{-s}. \tag{38}$$

This function has the form depicted in Figure 1, wherein, unlike the airfoil cases, a strong initial "overshoot" in lift force is seen to occur. This form has a trend that is typical of a number of bluff sections.

Employing equation (38) in equation (37) leads in this case to definition of the admittance function

$$|\chi_B|^2 = F_B^2 + G_B^2, \tag{39}$$

where F_B and G_B are defined by equations (17) and (18) when F is replaced by F_B, G by G_B and the current set of the parameters a, b, c and d is as employed in equation (38). The function $|\chi_B|^2$ is plotted in Figure 3. An interesting feature of this admittance function is that it is the first encountered in the present review that exceeds unit value in the range shown. That an admittance for a bluff body can exceed unity, however, is not rare, as often observed experimentally (Kumarasena 1989; Larose & Livesey 1997).

7. ADMITTANCES OF REPRESENTATIVE BRIDGE GIRDERS

Equation (36) explicitly defines the aerodynamic admittance associated with a bluff section for which C'_L is the associated static vertical lift curve slope and KH_1^*, KH_4^* are the accompanying flutter derivatives. Thus, for analyses employing admittances in the frequency domain, prior acquisition of the associated indicial function is not a necessary step.

Some representative bridges for which flutter derivative data are available to the writer are the Tsurumi Fairway (cable-stayed) and the Golden Gate, Akashi Kaikyo and Carquinez Strait (project) suspension spans. The experimentally measured lift curve slopes $(C'_L)_m$ at zero angle of attack, and drag coefficients C_D, for these bridges are listed in Table 1 below. The effective vertical lift curve slope C'_L under these conditions is $(C'_L)_m + C_D$.

The flutter derivatives H_i^*, A_i^* appearing in equations (28) and (29) are functions of $(K = 2k)$ typically made available in graphical form where the horizontal scale is $U/nB = 2\pi/K$. Figure 4 depicts an example of the functions $H_1^*, \ldots, H_4^*$ as presented by Katsuchi *et al.* (1998) for the Akashi Kaikyo Bridge. From such information and the values of C'_L in Table 1, the associated aerodynamic admittances [equation (36)] for vertical motions of four different bridges have been derived as listed in Table 2. Since the experimentally determined flutter derivatives are usually not practically needed or well defined beyond values of the abscissa $U/nB = \pi/k = 25$, it is not possible to assert with certainty what experimental values of $|\chi(k)|^2$ hold as $k \to 0$, although the likelihood is a convergence toward unity since this represents the nonoscillatory, steady state. It may particularly be noted in these examples that the form of the associated aerodynamic admittance is strongly dependent on the geometric shape of the body section in question.

TABLE 1

Bridge lift slope and drag coefficients

Location	Deck type	$(C'_L)_m$	C_D	C'_L
Tsurumi	(faired box)	3·370	0·104	3·474
Akashi	(open truss)	1·198	0·421	1·619
Golden Gate	(open truss)	2·865	0·350	3·215
Carquinez	(faired box)	4·501	0·144	4·645

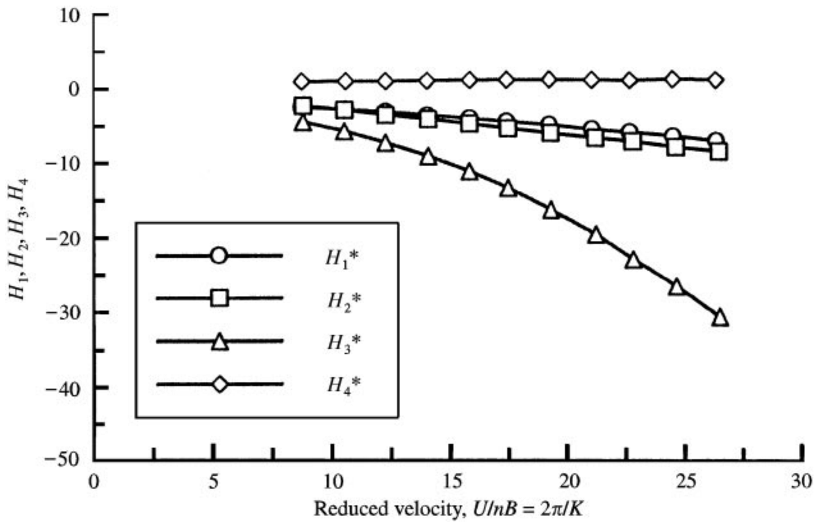

Figure 4. Akashi Strait flutter derivatives for vertical motion.

TABLE 2

Lift admittances of airfoil and bridges

π/k	k/π	Theodorsen $F^2 + G^2$	Sears $F_S^2 + G_S^2$	Tsurumi	Akashi	Golden Gate	Carquinez
∞	0	1	1	(1)	(1)	(1)	(1)
20·00	0·05	0·63	0·56	0·93	1·10	1·07	0·83
10·00	0·10	0·48	0·39	0·93	1·10	0·88	0·73
5·00	0·20	0·33	0·24	0·93	1·10	0·91	0·58
3·33	0·30	0·29	0·18	0·93	1·10	1·00	0·38
2·50	0·40	0·28	0·14	0·93	1·10	1·93	0·32

In Table 2 the Theodorsen and Sears admittance functions are included for comparison. The admittances of this table are plotted in Figure 5. Sketches of the cross-sections of the associated bridges appear in Figure 6. Of the bridges, the Carquinez results appear coincidentally to follow somewhat the trends of the Sears and Theodorsen admittances. Of the admittances listed in Table 2 only the airfoil-linked Sears is directly associated with vertical gust penetration. The others ensue from a related conceptual phenomenon: relative structure–fluid vertical motion. To-date few experimental studies have been made that duplicate, for a bluff body, the conception underlying the Küssner/Sears airfoil gust entry. A different viewpoint will next be explored in the bluff-body case.

8. ESTIMATING VERTICAL BUFFETING FORCES

In the design of long-span flexible bridges the problem of gust-induced lift on the deck girder of long-span flexible bridges is one of recurring interest. If u and w are, respectively, horizontal and vertical components of wind gusting, a commonly employed quasi-steady

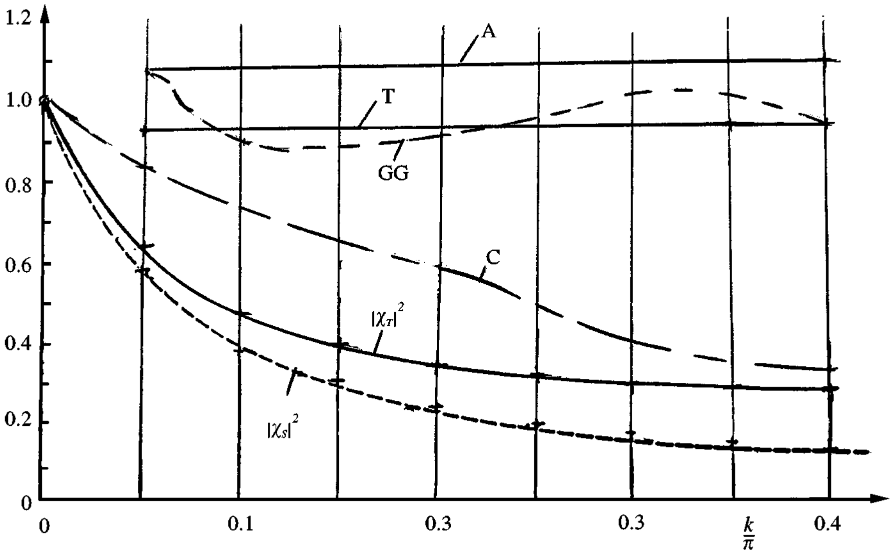

Figure 5. Unmodified aerodynamic admittances [equation (36), Table 2]. T = Tsurumi Fairway; A = Akashi Strait; GG = Golden Gate; C = Carquinez Strait.

first estimate of the lift on a section of width B is given by

$$L_b = \frac{1}{2}\rho U^2 B\left[2C_L\frac{u}{U} + C'_L\frac{w}{U}\right]. \tag{40}$$

For varying wind velocities, u, w, it is not certain that the steady-force coefficients C_L, C'_L remain valid. Hence an improved formulation is sought.

The concept employed in the vertical gust analysis of a bridge structure is not that the bridge "enters" (or is progressively enveloped by) a gust of infinite extent, as in the airfoil case, but that fixed-location, time-varying vertical gusting takes place at a point directly beneath the deck section. If the details of fluid–structure interaction due to a vertical gust $w(t)$ are considered to be the same as that occurring when the structure undergoes the identical relative motion in a vertical velocity $\dot{h}$, then from consideration of the flutter forces, an appropriate dynamic estimate for the lift slope C'_L will be

$$|C'_L| = K[H_1^{*2} + H_4^{*2}]^{1/2}, \tag{41}$$

as has been argued earlier [equation (36)].

This perception assumes that the vertical gust front completely extends across the width B of the structure. Since it may not do so in fact, the conceptual picture can be aided by further information on the nature of vertical gusts in the earth's atmosphere. The along-wind coherence of vertical gusting at fixed along-wind points in the atmospheric boundary layer of the earth is defined from measurements by Panofsky & Dutton (1983) via the form

$$\mathrm{coh}(w) = \exp\left[-c_0\frac{\sigma_w}{U}\left(\frac{n\Delta x}{U}\right)\right], \tag{42}$$

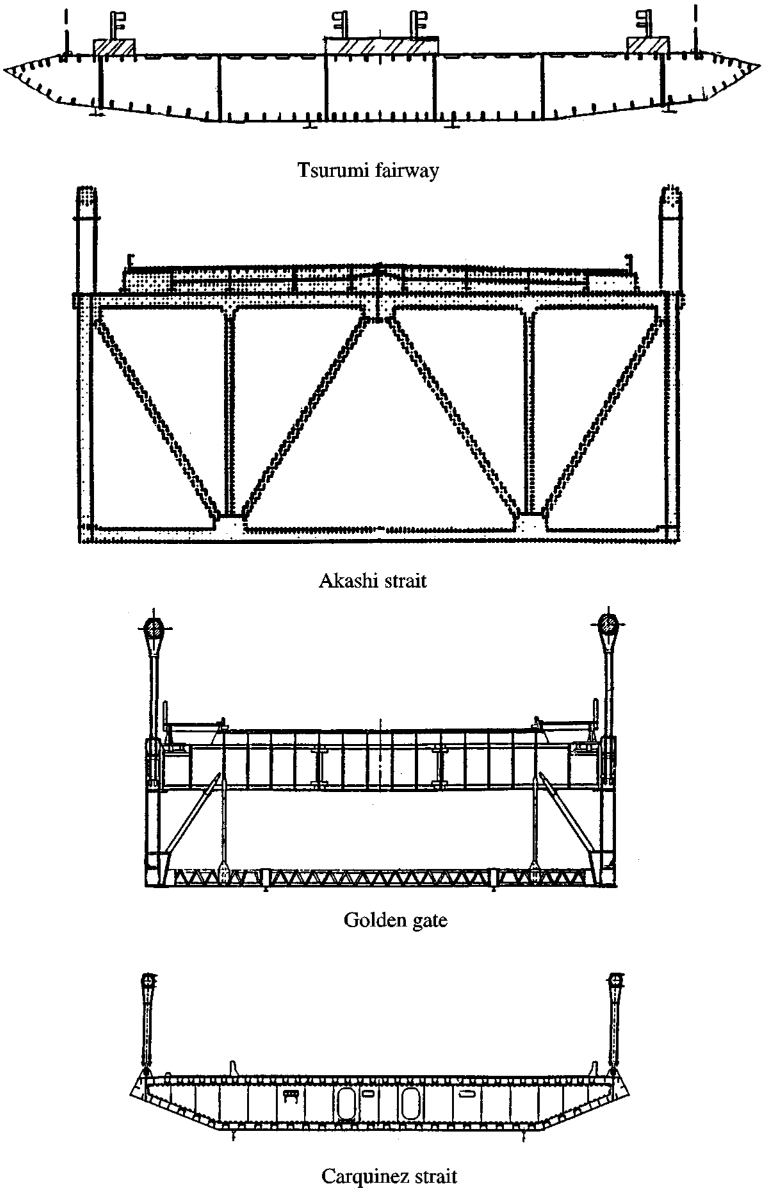

Figure 6. Four representative bridge sections.

where, in the atmospheric boundary layer, $c_0 = 60$, σ_w/U is the intensity of vertical turbulence, n is frequency and Δx is the along-wind separation of two reference points.

For convenience, let a new constant c be defined by

$$c = c_0 I_w \frac{nB}{U}, \tag{43}$$

where $I_w = \sigma_w/U$ is the intensity of vertical gusting. Important bridge vibration frequencies n fall in the range of 0·1–0·5 Hz. The ratio of vertical-to-horizontal wind spectra in this

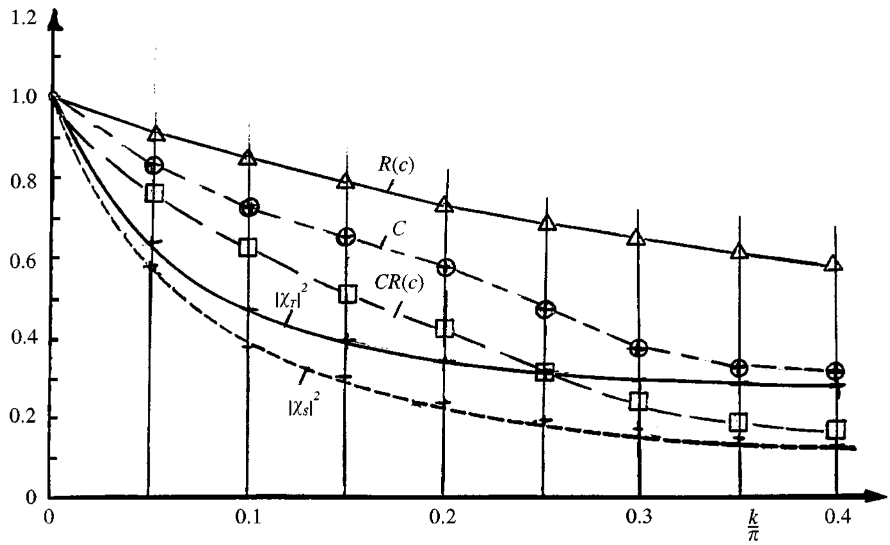

Figure 7. Representative longitudinal coherence factor $R(c)$ and both unmodified [C, equation (36)] and modified [$C \times R(c)$] aerodynamic admittance functions for the Carquinez Strait bridge deck cross section.

frequency range may be taken as $S_w/S_u \simeq 0{\cdot}6$, and a typical horizontal turbulence intensity may be 10%. Therefore, a value $c = 5nB/U$ will be a representative estimate for illustrative purposes.

The power spectrum S_L acting on the deck of width B of vertical lift per unit span may then be estimated via an along-chord coherence integral [cf., for example, Scanlan & Jones (1990)] as in the following form:

$$S_L = \left[\frac{1}{2}\rho UBC'_L\right]^2 \int_0^B \int_0^B S_w(n) \exp\left[-c_0 \frac{\sigma_w}{U} n|x_1 - x_2|\right] \mathrm{d}x_1\, \mathrm{d}x_2. \tag{44}$$

In view of the following well-known coherence result

$$R(c) = \int_0^1 \int_0^1 \mathrm{e}^{-c|\xi-\eta|}\, \mathrm{d}\xi\, \mathrm{d}\eta = \frac{2}{c^2}(c - 1 + \mathrm{e}^{-c}), \tag{45}$$

the chordwise-integrated lift spectrum at a spanwise deck-girder position takes the spectral form

$$S_L = [\tfrac{1}{2}\rho UBC'_L]^2\, S_w R(c). \tag{46}$$

In this latter form, C'_L may be estimated from equation (41), so that a net modified admittance function for lift due to vertical gusting can be estimated as

$$|\chi_{L'}|^2 = \frac{K^2}{(C'_L)^2}\,[H_1^{*2} + H_4^{*2}]\, R(c). \tag{47}$$

As an example, the functions $K^2[H_1^{*2} + H_4^{*2}]/(C'_L)^2$ and $R(c)$ as well as their product $|\chi_{L'}|^2$ are plotted in Figure 7 for the Carquinez Strait Bridge with $c = 5nB/U$. The above illustrative expression for lift due to vertical gust effects has not previously appeared in the literature to the writer's knowledge. A treatment of the effect on moment can be carried out by analogous means.

9. CONCLUSIONS

In this paper, some examples of aerodynamic admittance are examined, with thin airfoil theory first offering some perspective on its nature, particularly on the close similarity between the effects associated with vertical gust entry and vertical structural motion. These analogical observations, carried over to bluff-bodies, offer a paradigm for estimating the aerodynamic admittance for a bluff-body subject to transverse gusting. This is based on information inherent in measured flutter derivatives plus available data on the coherence of the afferent vertical components of turbulence.

As sound calculational approaches are needed in the wind response assessment of important bridges in their design stages, the point of what is discussed in this paper is to attempt a more accurate assessment, for the class of structures addressed, of across-flow buffeting loads than has typically been employed in past applications.

While theory of the thin airfoil remains a valuable guide as to *conceptual* steps, because of its definitional constraints it does not offer directly transferable results for bluff-bodies. This is pointedly true, for example, for thin-airfoil flutter and gust penetration results, such as the Theodorsen and Sears admittances; and it is also true for quasi-steady formulations based on standard static lift and drag coefficients, which imply constant unit admittances. The approach outlined in this paper offers an alternative rationale, for bluff sectional forms, that takes into account the experimentally based frequency dependence of the forces investigated. In this scheme the proposed admittances, based upon measurements, are less limited by *a priori* conceptualizations.

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APPENDIX: NOTATION

a	dimensionless distance to airfoil pivot, or lift function parameter
$A_i^*(K)$	torsional flutter derivative, function of K
b	lift function parameter
B	deck or airfoil width (chord)
c	lift function parameter or coherence coefficient
$C(k)$	Theodorsen function
C_L	lift coefficient
C_D	drag coefficient
d	lift function parameter
D	drag force per unit span
$F(k)$	real part of Theodorsen function
$F_B(k)$	real part of bluff-body admittance function
$F_S(k)$	real part of Sears admittance function
$G(k)$	imaginary part of Theodorsen function
$G_B(k)$	imaginary part of bluff-body admittance function
$G_S(k)$	imaginary part of Sears admittance function
h	vertical degree of freedom
$H_i^*(k)$	vertical flutter derivative
k	reduced velocity, $K/2$
K	reduced velocity, $B\omega/U$
L	lift force per unit span
n	frequency (Hz)
s	dimensionless distance or time, $2Ut/B$
S	power spectral density
t	time
u	horizontal wind gust velocity
U	steady horizontal wind velocity
w	vertical wind gust velocity
α	angle of attack of wind
$\chi_B(k)$	complex bluff-body admittance function
$\chi_S(k)$	complex Sears admittance function
$\chi_T(k)$	complex Theodorsen admittance function
$\lvert\chi(k)\rvert^2$	(absolute value)^2, aerodynamic admittance
ρ	air density
σ	variable on range of s
Φ	bluff-body indicial function
$\phi(s)$	Wagner indicial function
Ψ	Küssner indicial function
ω	circular frequency of oscillation
$\overline{(\ )}$	Fourier transform of ( )